

# The calculation of magnetic induction in grain orientated ultra-thin silicon steel sheets

Gao Xiuhua · Qi Kemin · Qiu Chunlin ·  
Tian Yanwen

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**Abstract** In this work the magnetic induction in grain orientated ultra-thin silicon steel sheets (with a thickness of 0.08 mm) was calculated by employing the crystal orientation distribution function and the formula for the anisotropy of energy for a single crystal of cubic symmetry. The incomplete pole figures of {110}, {200} and {112} were measured and the corresponding orientation distribution function was determined. On the basis of the texture data and the corresponding magnetic anisotropy energy, the magnetic induction in the ultra-thin silicon steel sheet was calculated. The calculation results are in good agreement with the experimental measurement.

## Introduction

Macroscopic performance of textured material can be divided into two categories with respect to the influence of crystallographic direction. One is the single axis performance which is determined by one crystal direction (e.g., the magnetic property of soft magnetic material). The other is the three-axis performance which can be described by using the four-step tensors. Examples include elasticity, plasticity and yield stress. The magnetic behavior of grain-orientated thin silicon steel sheet is a typical example of the

single axis performance. Since the grain boundaries in this material do not exert a significant effect on the magnetic behavior, one may neglect the effect of grain boundaries to estimate the magnetic performance by using the orientation distribution function.

Grain-oriented silicon steel sheet is a type of textured material. The macroscopic properties of grain-oriented thin silicon steel sheet are influenced by both the anisotropy of the single crystal and the grain orientation distribution function. If the anisotropy of the single crystal and the polycrystalline orientation distribution functions are known, one can theoretically determine the macroscopic properties of polycrystalline material.

The calculation of macroscopic properties of polycrystalline material based on single crystal anisotropy and texture data is important in the field of quantitative analysis of textured materials [1–9]. Texture is generally considered to be the most important factor governing the macroscopic performance of silicon steels [10]. Since the development of the orientation distribution function, it is possible to obtain the magneto-crystalline energy of textured polycrystalline materials by averaging the formula for the crystal anisotropy and the crystal orientation distribution function. Hutchison et al [11] first calculated the magnetism torque curve using the orientation distribution function of non-orientated silicon steel sheet. Morris et al [12] then calculated the magnetism torque curve, the iron loss and the magnetic susceptibility of non-orientated silicon steel sheet. Szpunar et al [13–14] directly calculated the magnetization curve of orientated silicon steel based on the single crystal magnetization theory.

Magnetic performance is the most important attribute of silicon steels. Understanding the relationship between macroscopic performance and microscopic characteristics is necessary in order to improve the magnetic properties of

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G. Xiuhua (✉) · Q. Kemin · Q. Chunlin  
The State Key Laboratory of Rolling and Automation,  
Northeastern University, Shenyang, 110004, China  
e-mail: gaoksiuhua@126.com

T. Yanwen  
School of Materials and Metallurgy, Northeastern University,  
Shenyang 110004, China

silicon steels. The theoretical calculation of magnetic properties based on crystallographic texture is also very important for practical applications. It can not only forecast the macroscopic performance of material but also can reduce the need for complex testing. In addition, it would enhance material design and would provide the industry the possibility of on-line monitoring material performance.

### Basic principle

Rolled and annealed polycrystalline materials generally exhibit magnetic anisotropy. The material texture is an important attribute of soft magnetic performance. With the development of the quantitative texture analysis of materials, the magnetic anisotropy from a series of expansion coefficient of texture has been calculated.

One may assume a single crystal has magnetic property  $F(\theta, \varphi)$ . In a crystallographic direction of  $(\theta, \varphi)$  ( $\theta$  and  $\varphi$  is the polar angle and the argument in crystal coordinates, respectively), its progression expression is [15–16]:

$$F(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{n=-l}^l B_{ln} P_l^n(\cos \theta) e^{-in\varphi} \quad (1)$$

where  $B_{ln}$  is the series coefficient and  $P_l^n(\cos \theta)$  the associated Legendre polynomial. For the material with cubic structure, as a result of the symmetry, Eq. 1 becomes the following:

$$F(\theta, \varphi) = B_{00} + B_{40}[P_4(\cos \theta) + 2a_{44}P_4^4(\cos \theta) \cos 4\varphi] + B_{60}[P_6(\cos \theta) + 2a_{64}P_6^4(\cos \theta) \cos 4\varphi] + \dots \quad (2)$$

where  $B_{00}$ ,  $B_{40}$ ,  $B_{60}$  are the property parameters of single crystal and  $a_{ln}$  is the coefficient between  $W_{lmn}$  and  $W_{lm0}$  ( $a_{44} = 0.5976143$ ,  $a_{64} = -1.870828$ ).

On a direction  $(\beta, \alpha)$  of the polycrystalline material, the series expression of axes density  $t_{\beta\alpha}(\theta, \varphi)$  of inverse pole figure on the rolling plane is:

$$t_{\beta\alpha}(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{n=-l}^l T_{ln}(\beta, \alpha) P_l^n(\cos \theta) e^{-in\varphi} \quad (3)$$

where  $T_{ln}(\beta, \alpha)$  is the series coefficient. For any direction  $(\beta, \alpha)$  of a cubic polycrystalline material ( $\beta$  and  $\alpha$  is the polar angle and the argument in sample coordinates OABC respectively), the series expression of axes density  $T_{ln}(\beta, \alpha)$  of inverse pole figure is:

$$T_{ln} = (-1)^n 2\pi W_{l0n} \quad (4)$$

The series coefficient  $T_{ln}(\beta, \alpha)$  of axis density in inverse pole figure is

$$T_{ln}(\beta, \alpha) = (-1)^n \cdot 2\pi \cdot \left(\frac{2}{2l+1}\right)^{\frac{1}{2}} \sum_{p=-l}^l Z_{lp0}(\cos \beta) e^{-ip\alpha} W_{lpn} \quad (5)$$

where  $Z_{lp0}(\cos \beta)$  is the augmented Jacobian polynomial and  $W_{lpm}$  is the series coefficient of orientation distribution function.

If the texture is known, the average value of the physical property  $\bar{F}(\beta, \alpha)$  along the direction  $(\beta, \alpha)$  of the specimen can be determined as the following:

$$\bar{F}(\beta, \alpha) = \int_0^{2\pi} \int_0^{\pi} F(\theta, \varphi) t_{\beta\alpha}(\theta, \varphi) \sin \theta d\theta d\varphi \quad (6)$$

After substituting Eqs. 2 and 3 into Eq. 6, the following formula is derived:

$$\bar{F}(\beta, \alpha) = B_{00} + 2\pi \sum_{l=4}^{\infty} \sum_{n=-l}^l (-1)^n B_{ln} T_{ln}(\beta, \alpha) \quad (7)$$

where  $l$  is in the range from 4 to  $\infty$ .

We are interested in the anisotropy of energy  $\bar{F}(\alpha)$  in the sheet, i.e., for  $\beta = 90^\circ$ . Due to the cubic structure, Eq. 7 may therefore be written into:

$$\begin{aligned} \bar{F}(\alpha) = & B_{00} + 4\pi^2 \left(\frac{2}{9}\right)^{\frac{1}{2}} (1 + 2\alpha_{44}^2) [P_4(0)A_{400} \\ & + 2P_4^2(0) \cos 2\alpha A_{420} + 2P_4^4(0) \cos 4\alpha A_{440}] B_{40} \\ & + 4\pi^2 \left(\frac{2}{13}\right)^{\frac{1}{2}} (1 + 2\alpha_{64}^2) [P_6(0)A_{600} \\ & + 2P_6^2(0) \cos 2\alpha A_{620} + 2P_6^4(0) \cos 4\alpha A_{640} \\ & + 2P_6^6(0) \cos 6\alpha A_{660}] B_{60} + \dots \end{aligned} \quad (8)$$

And simplify Eq. 8 as:

$$\bar{F}(\alpha) = B_{00} + B_{40}T^4(\alpha) + B_{60}T^6(\alpha) + \dots \quad (9)$$

where  $T^l(\alpha)$  is the coefficient which is related to texture.

Conversely,  $\bar{F}(\alpha_i)$  may be derived from the experimentally measured values at different  $\alpha$  in the tested sheet by means of a least squares method.

$$\begin{aligned} \sum_{i=1}^n \frac{\partial}{\partial B_{j0}} [B_{00} + B_{40}T^4(\alpha_i) + B_{60}T^6(\alpha_i) \\ + \dots - \bar{F}(\alpha_i)]^2 = 0 \end{aligned} \quad (10)$$

If the texture coefficients and the single crystal anisotropy constants  $B_{00}$ ,  $B_{40}$ ,  $B_{60}$  are known,  $\bar{F}(\alpha_i)$  may be calculated according to Eq. 10. This formula is used to predict the magnetic properties with texture data.

### Experimental procedures

The silicon steel sheet with (110)[001] texture and 0.3 mm thickness was used as the starting material. The chemical composition of the silicon steel sheet is (in wt%): C 0.07, Si 3.15, Mn 0.06, S 0.02, Cu 0.17, N 0.003, P 0.02. After the surface glass film was removed by pickling, the sheets were cold-rolled to 0.08 mm in thickness by cross shear rolling with a mismatch speed ratio of 1.17 using  $\phi$  90 mm/ $\phi$  200 mm  $\times$  200 mm four high rolling mill [17]. MgO was adopted as the coating material. Specimens were recrystallized at 840 °C for 6 h in a pure hydrogen atmosphere. The magnetic induction of the ultra-thin silicon steel sheet in different angles of 0°, 15°, 30°, 45°, 60°, 75° and 90° were measured. The magnetic properties were measured by using a single sheet tester [18]. The recrystallized texture of the thin sheets was measured by means of X-ray diffraction. The incomplete pole figures of {110}, {200} and {112} were obtained and the corresponding orientation distribution function was calculated using the two-step method [19].

### Results and discussion

On the basis of the anisotropy of a single crystal and the texture data, the magnetic properties of silicon steel were calculated [20].

It was found that the magnetic properties of the silicon steel sheet specimens displayed a strong dependence on the measurement direction angle  $\alpha$  (The  $\alpha$  angle is between the observed direction and the rolling direction on sheet plane). The magnetic inductions at 800 A/m  $B_8$  of thin silicon steel sheet along different  $\alpha$  angles were shown in Fig. 1. For  $\alpha$  less than 45°, the magnetic induction becomes lower with increasing  $\alpha$  value. For  $\alpha$  above 60°, it becomes higher as  $\alpha$  increases. As one can see from Fig. 1, the maximum magnetic induction is found at the angle  $\alpha = 0^\circ$ . Namely, the rolling direction is the easiest magnetization direction.

In the present study, incomplete pole figures of {110}, {200} and {112} were used to calculate texture coefficients. The series expansion function of the distribution function was truncated at  $l = 16$ . Figure 2 presents the constant  $\Psi$  sections of the ODFs in the silicon steel sheets which were rolled with MSR 1.17 and then annealed at 840 °C for 6 h in pure hydrogen atmosphere. As shown in Fig. 2, a strong {110} <001> texture was present in the

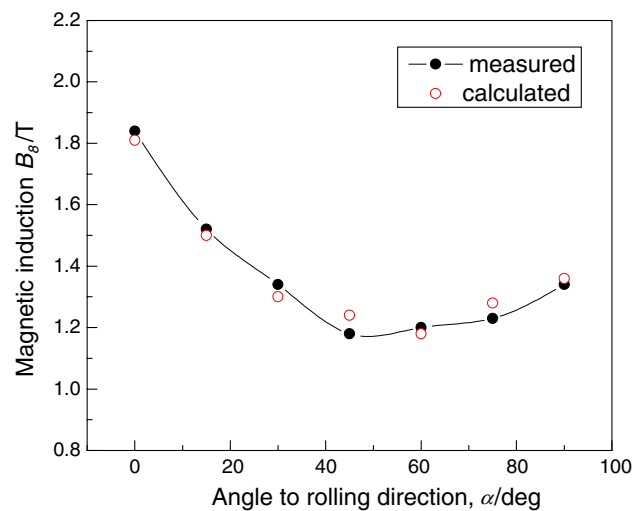


Fig. 1 A comparison between the calculated and the measured magnetic induction at different directions

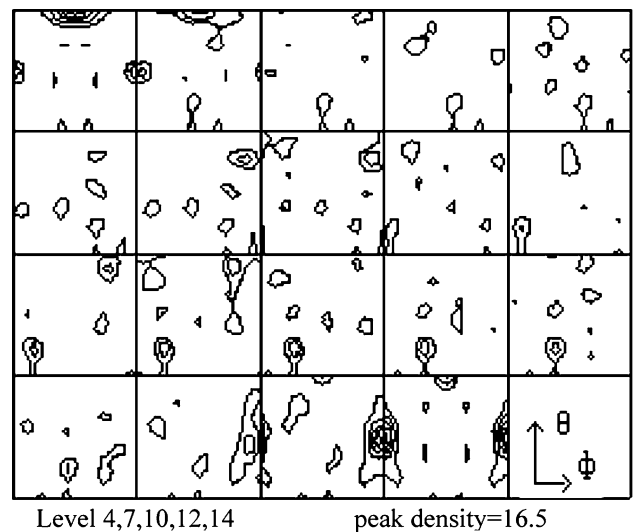


Fig. 2 Constant  $\Psi$  sections of ODFs of grain-oriented silicon steel thin sheet

surface of oriented silicon steel thin sheet. The maximum intensity is 16.5 units. The volume fraction of {110} <001> orientation is 52.1%. A slight texture was also observed along the {001} <110> direction.

According to Eq. 10, the average magnetic properties  $\bar{F}(\alpha_i)$  of thin silicon sheets at any angle  $\alpha$  can be calculated. Based on Friedel’s law, magnetic properties are considered to be a one-dimension physical property, which is related to one dimensional orientation. L in Eq. 6 is generally determined by low step progression. Then, in Eq. 8 the series expansion was truncated up to  $l = 8$ . The  $B_{00}$ ,  $B_{40}$ ,  $B_{60}$  and  $B_{80}$  values to magnetic inductions at 800 A/m  $B_8$  are given in Table 1. Table 1 indicates that  $B_{80}$  is smaller

**Table 1** The single crystal property coefficients of thin silicon steel sheet

| $B_{00}$ | $B_{40}$ | $B_{60}$ | $B_{80}$ |
|----------|----------|----------|----------|
| 1.4174   | -0.1056  | 0.00943  | 0.00027  |

than  $B_{00}$ ,  $B_{40}$  and  $B_{60}$ . Therefore, one may ignore its influence on magnetic properties. After  $l > 6$ ,  $B_{ln}$  has been approximated to zero, and, therefore, the single crystal anisotropy constants are only  $B_{00}$ ,  $B_{40}$ , and  $B_{60}$ .

The calculation results were compared with the experimental data in Fig. 1. It is clear that there is a good agreement between the theoretical and experimental data in the sheet plane, with a high relation coefficient of 0.98 and the relative error between calculated and experimental data less than 5%.

## Conclusions

The magnetic induction of ultra-thin grain-oriented silicon steel sheets can be predicted from the texture data. The prediction is based on the mathematical model on the basis of the texture measurement and the corresponding orientation distribution function. The magnetic induction along various directions in the plane of the sheet was calculated. The calculated data displayed a good agreement with the experimental data.

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